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EFFECT OF DISPERSION IN THE 3-D CONDENSED NODE TLM MESH

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ABSTRACT

The general dispersion equation, derived for the 3D-TLM condensed node mesh is presented and compared to dispersion of the FD-TD and the expanded node TLM schemes. Spurious mode propagation, as predicted by the dispersion equation for the condensed node, is also addressed. Analysis of dispersive effects is essential to assessing abnormalities in time domain simulations.

INTRODUCTION

The 3D TLM condensed nodes, developed by P. Johns, are arranged in a cubic lattice structure interconnected by dispersionless transmission lines.[1] As with any numerical method that relies on spatial and time sampling, the condensed node has undesired dispersion and spurious solutions associated with it. The dispersion characteristics of the condensed node are superior to that of the FD-TD scheme developed by Yee [2] and the expanded TLM node [3], as demonstrated by the general dispersion equation for the condensed node.[4]

Plane waves excited in the 3D mesh, that have spatial wavelengths that are only a few node spacings in length, suffer significant dispersion effects.[4] If the spatial wavelength is sufficiently short, propagating spurious solutions can be excited.[5]

GENERAL DISPERSION RELATION

The general dispersion relation for the condensed node mesh is derived by applying Floquet's theorem to an infinite three dimensional mesh. A plane wave solution is assumed with component propagation constants of k_x, k_y and k_z in the **x**, **y**, and **z** directions respectively. In its final form, the dispersion relation for the condensed node is given as [4]

$$\det(\boldsymbol{I}-\boldsymbol{TPS})=0 \qquad (1)$$

I is a 12 by 12 identity matrix, **T** is given by

$$\boldsymbol{T}=e^{-jk_o d}\boldsymbol{I}$$

where "d" is the node lattice spacing and ko is the propagation constant along the interconnecting transmission lines. **S** is the scattering matrix of the condensed node.[1] The elements of **P** are 0 except for;

$$P_{1,12} = P_{5,7} = e^{j k_y d}$$

$$P_{2,9} = P_{4,8} = e^{j k_z d}$$

$$P_{3,11} = P_{6,10} = e^{j k_x d}$$

$$P_{7,5} = P_{12,1} = e^{-j k_y d}$$

$$P_{8,4} = P_{9,2} = e^{-j k_z d}$$

$$P_{10,6} = P_{11,3} = e^{-j k_x d}$$

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Fig.1 displays results of the dispersion equation evaluated for a plane wave propagating in the y-z plane such that $k_x=0$. Note that the dispersion is zero along the y and z axis and that the maximum dispersion occurs for propagation along the diagonal y=z.

The general dispersion relation for the FD-TD 3D node is given by [6]

$$\sin(k_o d\frac{s}{2}) =$$

$$s^2(\sin^2(\frac{k_x d}{2})+\sin^2(\frac{k_y d}{2})+\sin^2(\frac{k_z d}{2})) \qquad (2)$$

where s is the stability factor. Fig. 2 shows the resulting family of dispersion curves for the case when s=½ where the FD-TD method is identical to the expanded 3D TLM formulation. Maximum dispersion occurs for propagation along the axis where the dispersion of the condensed node is zero. Minimum dispersion occurs along the diagonal $k_y = k_z$. The dispersion along this bearing is identical to the worst case dispersion of the condensed node. The dispersion for the FD-TD scheme can be improved slightly by choosing the maximum stability factor of $3^{-1/2}$.[4]

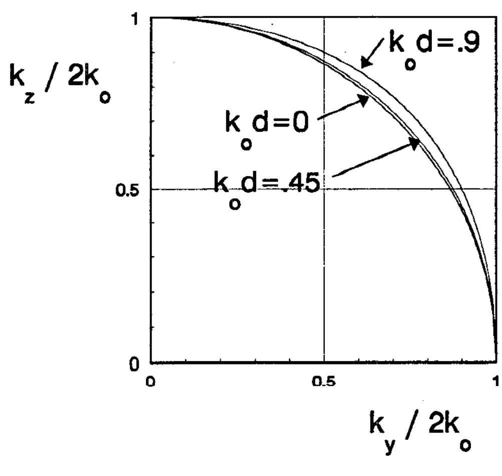

Fig.1 Plot of the dispersion characteristics for a condensed node TLM mesh supporting a plane wave propagating in the y-z plane.

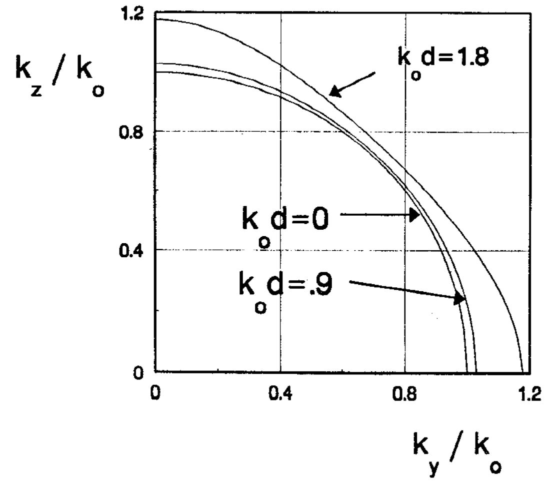

Fig.2 Plot of the dispersion characteristics for the FD-TD node with s=½ supporting a plane wave propagating in the y-z plane. (Note that k_o in the FD-TD plot is twice the k_o used in the corresponding TLM plot due to the velocity difference of a factor of 2.[3])

SPURIOUS SOLUTIONS

Consider an arbitrary excitation source in the x-y plane of an infinite 3D mesh. This source will excite an infinite number of evanescent and propagating plane waves. The propagation characteristics of each plane wave depends on the excitation frequency and the components of the transverse propagation vector, k_x and k_y. The plane wave falls into one of four regions as outlined in Fig.3. The first region for small $k_x d$ and $k_y d$ is the region of "physical propagating modes". Assuming an excitation frequency such that $k_o d$ is small relative to π, the boundary of this region is approximately circular, of radius $2k_o d$. The adjacent region is denoted "physical evanescent modes" characterized by a purely imaginary $k_z d$ that increases in magnitude with the modal index. Near the physical mode cutoff boundary, the imaginary part of $k_z d$ follows Eq.3 accurately provided $k_o d$ is reasonably small. However, as the spatial frequency is increased the imaginary part of $k_z d$ decreases rapidly and becomes negative infinity along the diagonal $k_x d+k_y d=\pi$. Crossing this line such that $k_x d+k_y d>\pi$, the imaginary part of $k_z d$ increases toward 0. Hence this region is denoted "spurious evanescent modes". The boundary between the spurious evanescent and propagating modes is a mirror image of the boundary separating the physical propagating and evanescent modes. The spurious propagating modes have a propagation constant of approximately

$$k_z d \approx \pi \pm \sqrt{2k_o d - (\pi - k_x d)^2 - (\pi - k_y d)^2} \quad (3)$$

For these modes $k_z d$ is real, indicating lossless *transmission.*

Spurious modes can be observed in a 3D TLM mesh cavity bounded by Dirichlet walls by exciting a mode with k_x, k_y and k_z close to π/d. As a demonstration, a cavity of 14 x 6 x 6 nodes was excited with the TE mode with $k_y d = k_z d = \pi$ and $k_x d = \pi(N_x - 1)/N_x$. The field distribution remained invariant with respect to time and oscillated with a period of 56.6Δt, which corresponded exactly to the spurious eigen-solution of Eq.1.

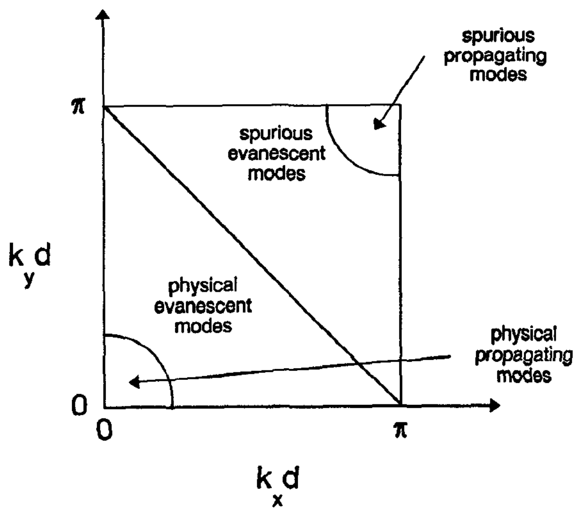

Fig.3 Spectral regions corresponding to physical and spurious plane wave mode propagation.

Taken from J. Nielsen, "Spurious modes of the TLM Condensed node formulation" IEEE Microwave and Guided Letters, Vol.1. No.8, 1991

CONCLUSION

In this paper, the general dispersion equation of the condensed node was addressed demonstrating superior dispersion characteristics in comparison to the FD-TD method. Attributes of the spurious modes can also be derived from the dispersion equation. There are as many spurious propagating modes as physically propagating modes. Hence spurious modes cannot be eliminated by temporal filtering but only through 3D spatial low pass filtering.

The eigenvector of the matrix, **I-TPS**, in the dispersion relation of Eq.1 corresponds to the incident voltage vector of the condensed node. Hence, the eigen-field quantities of any mode can be determined. An excitation field can then be written as a superposition of modes in the TLM mesh to determine errors in the TLM simulation due to dispersion and spurious modes.

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